A-1

1. Correct Option (a)

Solution:

$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$. Also , $\frac{T_1}{T_2} = \frac{N_2}{N_1}$ where $N_1 = 30$ and $N_2 = 36$

Therefore , $\sqrt{\frac{l_1}{l_2}} = \frac{N_2}{N_1}$ and $l_1 - l_2 = 22$ cm.

Solving the above two equations , we obtain , $l_1 = 72$ cm and $l_2 = 50$ cm

2. Correct Option : (b)

Solution : Waves are represented by

$y_1 = a\sin[2\pi(n-1)t]$, $y_2 = a\sin[2\pi nt]$ and $y_3 = a\sin[2\pi(n+1)t]$ respectively.

Superposition gives

$Y = y_1 + y_2 + y_3 = a(1 + \cos 2\pi t)\sin 2\pi nt = A \sin 2\pi nt$.

Where A is the amplitude.

Intensity (I) is proportional to A2 and I will be maximum when

Cos $2\pi t = 1$ i.e $t = 0, 2\pi, 4\pi, 6\pi$, etc. and t = 0, 1s , 2s , 3s, etc.

3. **Correct Option : (a)**

4. **Correct Option : (b)**

If the two masses be m ω^2 then the reduced mass $\mu = \frac{m}{2}$. So the the square of the frequency $\omega^2 = \frac{K}{\mu}$ where K is the spring constant. When one of the masses is stopped then the square of the new frequency

$\omega_n^2 = \frac{K}{m}$. So $\omega_n^2 = \frac{\omega 2}{2}$.

5. Correct Option : (b)

A simple harmonic oscillation can always written as $x = P\ sin(\omega\ t + \alpha)$. So the kinetic energy of the particle is

$$\frac{1}{2}m(\frac{dx}{dt})^2 = \frac{1}{2}mP^2\omega^2cos^2(\omega t + \alpha)$$

This can be written as

$$KE = \frac{mP^2\omega^2}{4} + \frac{m^2P^2\omega^2}{4}cos2(\omega t + \alpha)$$

This is nothing but a sinusoidal oscillation of frequency 2ω about $\frac{mP^2\omega^2}{4}$ reaching a minimum value 0 (never going to a negative value).

6. Correct option: (a)

7. Question Deleted

8. Correct option (d)

$$\frac{mL_{He}}{mL_w} = \frac{Pt_{He}}{PT_w}$$ Note that the two latent Heats are in different units.

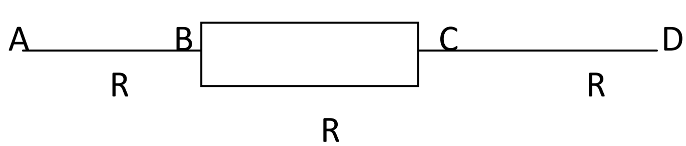
9 Correct option: (b)

$\underline{Using\ PV = nRT\ \ we\ get\ T = 240.7\ K\ \ The\ KE\ of\ one\ molecule\ of\ an\ oxygen\ at}$*om is*

given by $E = (3/2)\ \frac{R}{N}\ T = 1.5 \times 1.38 \times 10^{-23}\ 240.7J = 4.98 \times 10^{-21}\ J$

10. Correct option: (b)

Let us assume that each rod has a resistance of R then the equivalent circuit will be



The resistance of each AB, BC and CD is R. Thus the temp will be equally distributed along the three parts. So temp diff between AB is 60°C Between B & C is 60°C . So total drop is 120°C. So the temp of C is 80°C.

11. Correct option: (b)

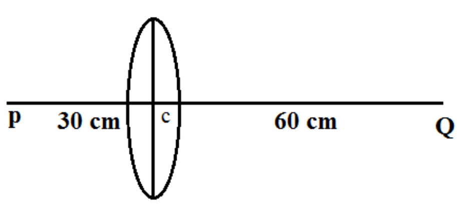
The total potential energy of the system is $3 * \frac{q^2}{a}$. When they are removed to infinity the P.E. is zero. So the K.E. = $\frac{3q^2}{a}$

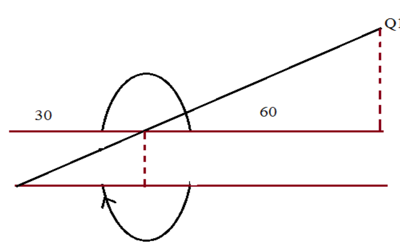
12. Correct option: (d)

The LDR current I is proportional to($V^2/R)/d^2$, where V is the voltage across the filament, R is its resistance and d is the distance between the source and the LDR So if both V and d are doubled the LDR current should have remained same but as R (of the bulb) also increases with temperature and hence current will be less than I.

13. Correct option: (a)

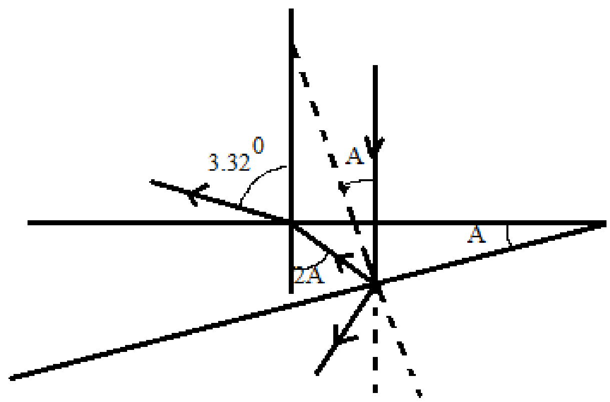
F=uv/(u+v)=30x60/(30+60)=20. d=(v/u)X5=10. The separation between the two images would be 10 +5 = 15 cm.





14. Correct option: (A)

The angle of refraction of a very thin prism A= 1^0. As the ray is incident normally on the first surface, the angle of incidence at the second surface is A and the ray is partly reflected from this surface, would make an angle 2A with the normal at the first surface. As the angles are very small so one can write refractive index n=3.32/2A =1.66. Again if D is the deviation suffered by the ray that emerges from the second surface, then , for small angles



$n= (A+D)/A$ i.e $D=(n-1)A=.66^0$

15. Correct option (b)

If R be the radius of curvature of the curved surface of the lens,f the focal length of the lens and n the refractive index of the material of the lens, then (1/R)(n-1)= 1/f i.e. R= 40(n-1).When the curved surface is silvered the ray reflected from the concave mirror would have converged at a distance of R/2, in absence of any solid transparent material. But as there is a material with refractive index n then R/2n = 7.5. From these two relations R=24cm and n=1.6 we get the result.

16. correct option (a)

It is apparent that first lens is concave and its focal length f is given by 1/f = 1/-15 – 1/-10
i.e. f= 30 cm . The second lens is convex so 1/v – 1/-10 = 1/-30 i.e. v=-7.5 cm.
Hence d=2.5.

17. Correct option: (d)

If c be the velocity of light in vacuum and v that in the medium then refractive index
$c/v = (2)^{1/2}$. Now if V is the voltage used to accelerate the electrons, each of mass m and charge e then
$V_e = (1/2)mv^2$. Using the values of the parameters it is found that V= 127.82 kV

.

18. Correct option (a)

$\frac{m_O}{m_{Ag}} = \frac{E_O}{E_{Ag}} \quad \Rightarrow \frac{0.8}{m_{Ag}} = \frac{16/2}{108} \quad or,\ m_{Ag} = 10.8$ gm

19. Correct option: (b)

$E_n = \frac{K z^2}{n^2}$ where K is a constant . For first exited state of Hydrogen (z =1) , n = 2.
$E_2 = \frac{K (1)^2}{2^2} = \frac{K}{4}$. For n_{th} state of Lithium (z = 3) we can write
$E_n = \frac{K (3)^2}{n^2} = \frac{9K}{n^2}$.
According to the question $E_2 = E_n$ or, $\frac{K}{4} = \frac{9K}{n^2} \quad \therefore n = 6.$

20. Correct option: (d)

Output of first OR gate is $A = (X + \overline{Y})$
Inputs of second OR gate is $\overline{X}$ and $\overline{A} = \overline{X}\ Y$
Output of second OR gate is $C = \overline{X} + \overline{A} = \overline{X} + \overline{X}\ Y = \overline{X}$
Final output is $Z = \overline{C} = X$

21. Correct option: (c)

Solution : Current through the 20 ohm resistance from right to left is
$i = \frac{2.5 - 1.0}{20 + 10} A = 0.05A$.
Therefore , - 1.0 – 0.05 x 20 = -0.5 + V_{AB} Thus , V_{AB} = - 1.5 V
system is infinity.

22. Correct option: (a

Solution : Since the compartments are thermally insulated , the total intyernal energy before and after opening of the valve will be the same.

The internal energy of a perfect gas is given by $u = \frac{PV}{\gamma - 1}$.

Therefore, $\frac{P_1V_1}{\gamma-1}+\frac{P_2V_2}{\gamma-1}=\frac{P(V_1+V_2)}{\gamma-1}$, where P and T are the pressure and temperature after opening the valve. This gives , $P=\frac{P_1V_1+P_2V_2}{(V_1+V_2)}$

Since the total no. of moles is the same before and after opening of the valve, we can write ,

$\frac{P_1V_1}{RT_1}+\frac{P_2V_2}{RT_2}=\frac{PV}{RT}$. This gives $T=\frac{PV}{\frac{P_1V_1}{T_1}+\frac{P_2V_2}{T_2}}=\frac{T_1T_2(P_1V_1+P_2V_2)}{(P_1V_1T_1+P_2V_2T_2)}$

23. Correct option: (b)

Let the currents through the 20 ohm (parallel) and 10 ohm (in series with the diode) be i_1 and i_2 respectively. The Kirchoffof's voltage equations are

$0.7 + 10\, i_2 - 20\, i_1 = =0$ ---------------------------- (1)

$20\, i_1 + 10\,(i_1 + i_2) - 10 = 0$ ----------------- (2)

Solving the two equations, We get $I_1 = 0.214$ A and $i_2 = 0.358$ A .

Thus , voltage across the 10 Ohm resistance in series with the diode = 0.358 x 10 = 3.58 V

And voltage across the 20 Ohm resistance = 0.214 x 20 = 4.28 V

24.Correct option (a)

$emf, e=\frac{d\phi}{dt}=2at+b$

current flowing,i $=\frac{e}{R}=\frac{2at+b}{R}=\frac{dq}{dt}$

Average emf = $=\frac{1}{\tau}\int_0^{\tau} edt = a\tau+b$

Total charge flowing = $=\int_0^{\tau} idt=\frac{a\tau^2+b\tau}{R}$

25. **Correct option: (b) .**

Current I $=\frac{12}{60}$ = 0.2A $\therefore$ $z_{coil}=\frac{27}{0.12}=135\ ohm$

Using three voltage formula we find

$cos\emptyset=\frac{33^2-27^2-12^2}{2x27x12}=\frac{216}{27x24}=\frac{1}{3}$ $\therefore r = zcos\emptyset = 135x\frac{1}{3}$ = 45

26. Correct option: (c)

Energy of oscillations is proportional to square if the frequency Hence the required work done is 4W – W = 3W

27. Correct option: (a).

The energy stored in the capacitor $\frac{1}{2}CV^2$is used up in heating the R.s. Stored Energy W=The energu stored in the capacitor $\frac{1}{2}CV^2$ is used up in heating the R.s. Stored Energy W=$\frac{1}{2}CV^2$=$\frac{1}{2}$ $8x10^{-6}x250^2$= 0.25 J hence Required Answer H=Wx375/(375+250) = 0.15 J

28. Correct option: (d)

Emf is induced in the two vertical wires only. Since the loop is closed the net emf $(B_1 – B_2)Lv$ Further, B due to the current I is B_1= $\frac{constant}{(x-\frac{a}{2})}$ and B_2= $\frac{constant}{(x+\frac{a}{2})}$ solving we get the result

29. Correct option: (c)

The two resistances R each form a potential divider circuit. Hence the capapcitor is charged to half the full voltage.

30. Correct option: (b)

Solution : Let the currents through the 20 ohm (parallel) and 10 ohm (in series with the diode) be i_1 and i_2 respectively. The Kirchoffof's voltage equations are

$0.7 +10\, i_2 -20\, i_1 = =0$ ---------------------------- (1)

$20\, i_1 + 10\, (i_1 + i_2) -10 = 0$ ----------------- (2)

Solving the two equations, We get $I_1 = 0.214$ A and $i_2 = 0.358$ A .
Thus , voltage across the 10 Ohm resistance in series with the diode = 0.358 x 10 = 3.58 V
And voltage across the 20 Ohm resistance = 0.214 x 20 = 4.28 V

31. Correct option: (b).

For a rigid diatomic molecule , the translational degrees of freedom is 3 and the rotational degrees of freedom is 2 and the total degrees of freedom is 5. Thus , the rotational kinetic energy of one mole is $\frac{1}{2}RT$ x 2=RT

The total kinetic energy of one mole $\frac{1}{2}RT$ x 5=$\frac{5}{2}$RT . And their ratio is 2/5.

32. Correct option: (a)

The pressure in the medium is $p = p_0 \cos \omega(t - \frac{x}{\mathrm{v}})$ where p_0 is given by $p_0 = \frac{Bs_0\omega}{\mathrm{v}}$. B is the bulk modulus of the medium, v is the velocity of the wave in the medium.

The average power across a unit surface is given by $W = \frac{Bs_0{}^2\omega^2}{2\,\mathrm{v}} = \frac{p_0{}^2}{2B}\mathrm{v}$

Since B and v are properties of the medium and not dependent on the wavelength and since in both cases are equal , we have $W_1 = W_2$.

33. Correct option: (d)

34. Correct option: (a) .

35. Correct option: (d).

36. Correct option is (c).

37. Correct option: (d)

38. Correct option: (b)

Electric field outside the sphere at a distance r from the centre of the sphere is E = $\frac{Q}{4\pi r^2}$. So $Q_{max} = 2 * 10^6 * 4\pi\epsilon_0 * r^2$ (For the field to be maximum on the surface of the sphere). Putting in the values we get Qmax = $5.6 * 10^{-7}$ Coulomb.

39. Correct Option: (a)

40. Correct option: (d)

A-2

41.Correct option: (a) & (c)

The apparent weight of a FLOATING object is zero. ∴ (a) is correct

The weight of the liquid displaced equals the weight of the floating object

∴ (c) is correct

42.Correct option: (a) & (d)

43.Correct option: (a) (b) & (d)

44.Correct option: (a) (b) (c) & (d)

The rate of change of angular momentum of a system of particles about the centre of mass is equal to the sum of the external torques about the CM whatever may be the state of motion of the CM. So all the answers are correct.

45.Correct Option : (a) & (c)

In vacuum the electric and magnetic fields and the propagation vector form a right handed coordinate system and they are in phase. So, the answers that satisfy these conditions are

46.Correct Option : (a) & (c)

47.Correct Option : (b) (c) & (d)

48. Correct Option (a) (b) & (d)

49Correct Option: (b) & (c)

50. Correct Option: (a) & (c)

Part B

`Q. 1 (a) $CdT = \frac{V^2}{R}$ dt - k (T – T_0) dt

R CdT = [V^2 - kR (T – T_0)] dt

= - kR (T – T_0 – $\frac{V^2}{kR}$) dt

$\frac{dT}{T - T0 - \frac{V^2}{kR}}$ = - $\frac{k}{C}$ dt .... integrating this eqn. we get

Ln (T – T0 $-\frac{V^2}{kR}$) = - $\frac{kt}{C}$ + B

T – T_0 – $\frac{V^2}{kR}$ = A $e^{-\frac{kt}{C}}$ but at t=0 ; T = T_0 that gives $A = -\frac{V^2}{kR}$

Hence T =T_0 + $\frac{V^2}{kR}$ - $\frac{V^2}{kR} e^{-\frac{kt}{C}}$ or T =T_0 + $\frac{V^2}{kR}$ (1 - $e^{-\frac{kt}{C}}$)

Therefore at t = $\frac{C}{k}$ the temperature of the conductor

will be T_0 + $\frac{V^2}{kR}$ (1 - $\frac{1}{e}$)

Q. 1 (b)

F = qE – q(E_0 – ax) = m ω where ω = acceleration

ω = $\frac{q}{m}$(E_0 – ax) ……….(1)

ω = $\frac{q}{m}$(E_0 – ax) = v.dv/dx; Integrating we get $\frac{v^3}{2} = \frac{q}{m}\ (E_0\ x - ax^{\ 2}) + A$

But v = 0 at x = 0 therefore constant A = 0

$$v = \frac{2qx}{m}\left(E_0\ - \frac{ax}{2}\right)$$

This shows that the particle again comes to rest when $\left(E_0\ - \frac{ax}{2}\right)$ = 0 (i.e) at

$x = \frac{2E_0}{a}$. Therefore the distance covered by the particle till it comes to rest again is $x = \frac{2E_0}{a}$. And the acceleration of the particle at this moment will be (-) $\frac{q}{m}$ E_0 from eqn (1) above.

2] (i) Isothermal process W_1 = RT ln ($\frac{V_2}{V_1}$) = 8.3 x 330 x ln(2) = 1890 J.

(ii) Isobaric process : W_2 = p.dv = p (V_2 – V_1) but V_2 = 2 V_1

W_2 = p V_1 = nRT_1 = 2737 J.

(iii) Adiabatic process : W_3 = $\frac{R}{\Upsilon-1}$ (T_1 – T_2) gives Type equation here.

W_3 = $\frac{8.3\ (320-242.5)}{0.4}$ = 1608 J

Hence W_2 is maximum (Isobaric process) and W_3 is minimum (Adiabatic process) and W_2 is greater than W_3 by 65 %

Q. 3. At time t =0 mass M_0 = M_c + M_s and if m is the mass at any instant

We have m = $\frac{dm}{dt}$ = -λ or m = M_0 –λt ……….(i)

Since there is no force on the carriage its momentum M_0V_0 is conserved

F v is the velocity of the carriage at any time t , we can write $mv = M_0 V_0$ or

$v = \frac{M0\ V0}{m} = \frac{M0\ V0}{M0 - \lambda t} = \frac{dx}{dt}$ integrating we get $x = \frac{M0\ V0}{-\lambda}$ ln $(M_0 - \lambda t) = A$.

But x = 0 at t = 0 hence $A = -\frac{M0\ V0}{\lambda}$ ln $(M_0 - \lambda t)$

Thus $x = \frac{M0\ V0}{\lambda}$ ln $\left(\frac{M0}{(M0 - \lambda t)}\right)$ .... (ii)

If the carriage becomes empty at $t = t_1$ eqn (i) gives $M_c = M_0 - \lambda t_1$ so that

$t_1 = \frac{M0 - Mc}{\lambda} = \frac{Ms}{\lambda}$ ....(iii)

Using eqn (ii) the distance at which carriage becomes empty is obtained as

$x_1 = \frac{M0\ V0}{\lambda}$ ln $\left(\frac{M0}{(M0 - Ms}\right) = \frac{M0\ V0}{\lambda}$ ln $\left(\frac{M0}{Mc}\right)$ and from expression for velocity above $v_1 = \frac{M0\ V0}{M0 - Ms} = \frac{M0\ V0}{Mc}$ this velocity then remains constant.

Q. 4.

At the point of incidence $\frac{\sin i}{sinr_1} = \mu = \frac{\sin e}{sinr_2}$ at the point of emergence

$$\mu = \frac{\sin i + \sin e}{sinr_1 + sinr_2}$$

using sin + sin formula, we get $\frac{\sin\frac{(i+e)}{2}cos\frac{(i-e)}{2}}{\sin\frac{r_1+r_2}{2}cos\frac{r_1-r_2}{2}} = \mu$

It can be easily shown that $(I + e) = (A + \delta)$ and $(r_1 + r_2) = A$

Hence the result.

At the minimum deviation I = e and $r_1 = r_2$ reducing the cosine terms to value 1.

The usual prism formula follows.

Q 5 (a)

Initial activity A = 20500 x 60 disintegrations/min

$\therefore \lambda N_0$ = 1230000 disint/min……………………(i)

If V is the volume of the blood, the activity of 1 ml blood aftertime t will be

$\frac{\lambda N}{V}$ = 20 disint / min (given)……………………(ii) Dividing (i) by (ii)

$\frac{N_0\ V}{N}$ = 61500

$\therefore V = 61500\ \frac{N}{N_0}$ = 61500. Exp ($-\ \frac{0.693\ x\ 5}{15}$) Solving we get

V = 48815 ml or 48.8 litres (this is not a realistic figure as the actual blood volume in a human body is around 5 litres)

Q 5 (b)

I = $\frac{E}{R}$ = $\frac{\frac{d\varphi}{dt}}{\lambda 2\pi\ (\ r_1 + r_{\ 2}\ )}$ = $\frac{\frac{d}{dt}[B\pi(r_2^2 - r_1^2\ )]}{2\pi\lambda(r_1 + r_2)}$

= $\frac{\frac{d}{dt}[\ \pi\ B_0\ (\ r_2^2 - r_1^2\ )\cos\omega t}{2\pi\ \lambda(r_1 + r_2)}$

= $\frac{[\ \pi\ B_0 \omega(\ r_2^2 - r_1^2\ )\sin\omega t}{2\pi\ \lambda(r_1 + r_2)}$ Therefore the amplitude of the current I_0 is given by

$$I_0 = \frac{B_0 \omega \left(r_2^2 - r_1^2 \right)}{2\,\lambda (r_1 + r_2)} = \frac{\omega\, B_0\, (\ r_2 - r_1\,)}{2\,\lambda}$$

$$= \frac{100\pi\ x\ 20\ x\ 10^{-3}\ x\ 10\ x\ 10^{-2}}{2\ x\ 0.1}\ \text{A} = 3.14\ \text{A}.$$