SOLUTION
THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
GAUSS CONTEST - FINAL - PRIMARY
CLASS - V & VI

Instructions:
1. Answer as many questions as possible.
2. Elegant and novel solutions will get extra credits.
3. Diagrams and explanations should be given wherever necessary.
4. Fill in FACE SLIP and your rough working should be in the answer book.
5. Maximum time allowed is THREE hours.
6. All questions carry equal marks.

1. Write down all the ten digit numbers whose digital sum is 2. (The digital sum of a number is the sum of the digits of the number. The digital sum of 4022 is 4 + 0 + 2 + 2 is 8). Find the sum of all the 10 digit numbers with digital sum 2.

Sol. The sum of 10 digit numbers is shown below :

	2	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	1
	1	0	0	0	0	0	0	0	1	0
	1	0	0	0	0	0	0	1	0	0
	1	0	0	0	0	0	1	0	0	0
	1	0	0	0	0	1	0	0	0	0
	1	0	0	0	1	0	0	0	0	0
	1	0	0	1	0	0	0	0	0	0
	1	0	1	0	0	0	0	0	0	0
	1	1	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1

2. The sum of the 3-digit numbers 35a and 4b7 is divisible by 36. Find all possible pairs (a, b).

Sol. $\Rightarrow$ 35a + 4b7 = 350 + a + 407 + 10b

$\Rightarrow 757 + 10b + a$

$\frac{757+10b+a}{36} \rightarrow \text{Remainder} = 1 + 10b + a$

$\therefore\ 1 + 10b + a = 36n$

For n = 1 $\quad 10b + a = 35 \rightarrow \boxed{b = 3, a = 5}$

n = 2 $\quad 10b + a = 71 \rightarrow \boxed{b = 7, a = 1}$

n = 3 $\quad 10b + a = 107$

b is single digit so n = 3 is not possible.

The possible pairs are (5, 3)(1, 7)

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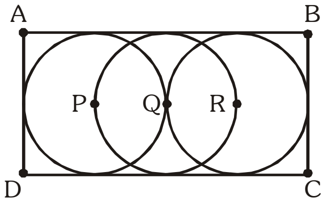
3. Three congruent circles with centres P, Q and R, are tangent to the sides of rectangle ABCD as shown. The circle with centre at Q has diameter 5 cm and passes through the points P and R. Find the area of the rectangle ABCD.

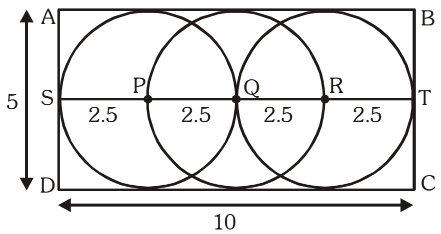


Sol. SP = PQ = QR = RT = 2.5 cm

Length = 4 × radius, breadth = 2 × radius

Length = 2 × 5 = 10 cm, Breadth = 5 cm

Area = 50 cm^2



4. A lucky year is one in which at least one date, when written in the form day/month/year, has the following property : The product of the month times the day equals the last two digits of the year. For example, 1944 is a lucky year because it has the date 11 / 4 / 44 where 11 × 4 = 44. From 1951 to 2000 how many years are not lucky? Give proper explanation for your answer.

Sol. As maximum available date is 31 therefore, years greater than 1950 having last two digits as prime numbers greater than 31 or multiple of prime numbers greater than 31 will not be lucky.

Also, 1958 and 1962 will not be lucky because 29/2/58 and 31/1/62 are not valid dates.

Also, 2000 is not a lucky year because date and months are natural numbers so their product will not be 00.

1953, 1958, 1959, 1961, 1962, 1967

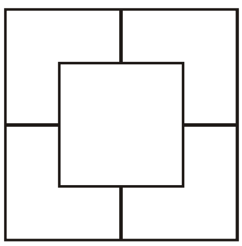
1971, 1973, 1974, 1979

1982, 1983, 1986, 1989

1994, 1997, 2000

Total not lucky no's are = 17

5. The area of each of the four congruent L-shaped regions of this 100-cm by 100-cm square is 3/16 of the total area. How many centimetres long is the side of the centre square?



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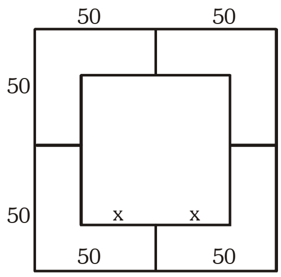
Sol. Let the side of inner square be 2x



$(100)^2 - (2x)^2 = 4 \times \frac{3}{16}(100)^2$

$\left(\frac{4}{16}\right)(100)^2 = (2x)^2$

$2x = 50$ cm

6. For any positive integer n, s(n) is the sum of the digits of n. What is the minimum value of $\frac{n}{s(n)}$ when (1) $10 \le n \le 99$ and (2) $100 \le n \le 99$.

Sol. (1) Maximum sum is possible when 9 will be there in n.
So, all n including 9 we will take.

$\frac{99}{18} = 5.5$, $\frac{89}{17} = 5.23$, $\frac{79}{16} = 4.9$, ... and so on.

Minimum value will be at $\frac{19}{10} = 1.9$

(2) Data given is wrong.

7. A 122 digit number is obtained by writing the 2 digit numbers 39 to 99 i.e., 39404142434445..... 96979899. You have to remove 61 digits from this number in such a way that the remaining digits in that order form the largest number possible. (for example in 15161718 if we remove the four 1's we get the number 5678, but if we remove 1, 5, 1 and the 1 after 6, we get 6718. This will be the largest number possible in the case). What will be the first 10 digits of the largest number obtained?

Sol. 9999777737

8. Given the numbers 2, 4, 8, 10, 14 and 16 : a % b is defined as the remainder when the ordinary product a.b. is divided by 18. Find the % product of every pair of these numbers including the product of a number with itself. Fill in the table given below.

(1) Find 2% 2% 2% ...%2, where we find the % product of fifteen 2's.

(2) Find 8% 8% 8%....%8 where we have ten 8's.

%	2	4	8	10	14	16
2						
4						
8				8		
10						
14		2				
16						

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Sol. Remainders in the table will be $\rightarrow 2\%\ 4 = 8,\ 2\%8 = 16$ and so on.

%	2	4	8	10	14	16
2	4	8	16	2	10	14
4	8	16	14	4	2	10
8	16	14	10	8	4	2
10	2	4	8	10	14	16
14	10	2	4	14	16	8
16	14	10	2	16	8	4

(a) $\rightarrow \dfrac{2^{15}}{18} = 8$ Remainder

(b) $\rightarrow \dfrac{(8)^{10}}{18} \rightarrow \dfrac{2^{30}}{18} = 10$ Remainder